

28 – Antiderivatives & Indefinite Integrals

Definition: Antiderivative

We say that F is an **antiderivative** for f if $F'(x) = f(x)$.

Theorem: General Antiderivative

If F is any one antiderivative for f , then *every* antiderivative for f has the form

$$F(x) + C$$

where C is an arbitrary constant. This is called the **general antiderivative**.

1. Find the general antiderivative of each of the following.

(a) $f(x) = x^4$

(b) $f(x) = 2 \cos x + 1$

Definition: Indefinite Integral

If F is any antiderivative for f , then we use an **indefinite integral** to represent the general antiderivative as follows:

$$\int f(x) dx = F(x) + C.$$

2. Find a formula for each of the following.

(a) $\int x^{-3} + \sec^2(x) dx$

(b) $\int \frac{2}{\sqrt{1-x^2}} dx$

Theorem: Antiderivatives of Power Functions

- $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ for $n \neq -1$
- $\int x^{-1} dx = \ln|x| + C$

3. Compute.

(a) $\int (5x^3 + \pi - \frac{1}{\sqrt{x}}) dx$

(b) $\int \sqrt{x}(1 + x^{\frac{5}{2}}) dx$

(c) $\int \cos(3x) dx$

4. Find a function $f(x)$ satisfying the following conditions.

$$f'(x) = \frac{3 + x^{-3}}{x}, \quad f(1) = 2$$