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03 – Introduction to Limits

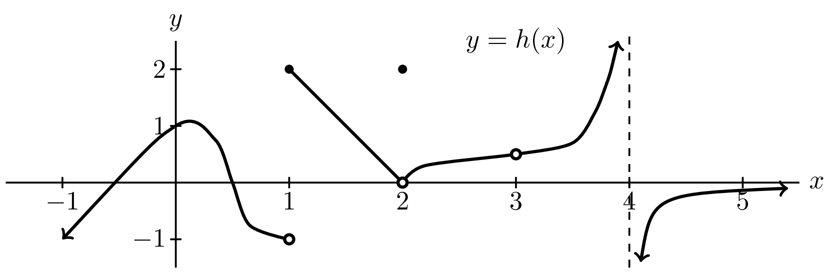
1. Suppose $f(x)$ is a mystery function for which some values are given (approximately) below.

x	-2	-1	-0.5	-0.1	0	0.1	0.5	1	2
$f(x)$	0.45	0.84	0.96	0.998		0.998	0.96	0.84	0.45

(a) Based on this data, what do you think is the value of $f(0)$?

(b) Look in the lower-left corner of page 2 to see what the mystery function is. Does this change your answer about $f(0)$? Explain.

2. Suppose the graph of a function $h(x)$ is given below. Find the value of each of the following below.



(a) $h(0) =$

(b) $h(1) =$

(c) $h(2) =$

(d) $h(3) =$

(e) $h(4) =$

So, there can be a **big** difference between the *actual* value of a function at a number a and what the values are *approaching* near a. The next definition captures what the values are approaching.

Definition: Limits (Informally)

We write $\lim_{x\to a} f(x) = L$ if the values of $f(x)$ can be made to stay arbitrarily close to L for all x sufficiently close to a, *but not equal to* a.

- $\lim_{x\to a} f(x)$ is asking where the outputs of the function *appear* to be going as x approaches a.
- $\lim_{x\to a} f(x) = L$ is read "the limit of $f(x)$ as x approaches a is L".

3. Answer the following questions about the functions f and h from above.

(a) $\lim_{x\to 0} f(x) =$

(b) $\lim_{x\to 0} h(x) =$

(c) $\lim_{x\to 1} h(x) =$

(d) $\lim_{x\to 2} h(x) =$

(e) $\lim_{x\to 3} h(x) =$

(f) $\lim_{x\to 4} h(x) =$

Definition: One-Sided Limits (Informally)

From the left: only considering x-values **less than** a, we define $\lim_{x\to a^-} f(x) = L$.

From the right: only considering x-values **greater than** a, we define $\lim_{x\to a^+} f(x) = L$.

4. Answer the following questions about the functions f and h (from the previous page).

(a) $\lim_{x\to 0^-} f(x) =$

(b) $\lim_{x\to 1^+} h(x) =$

(c) $\lim_{x\to 1^-} h(x) =$

(d) $\lim_{x\to 2^+} h(x) =$

(e) $\lim_{x\to 2^-} h(x) =$

(f) $\lim_{x\to 4^-} h(x) =$

5. Investigate $\lim_{x\to 0^+} \sin\left(\frac{\pi}{2x}\right)$ by following the steps below.

(a) Fill in the table below, and use it to make a guess about $\lim_{x\to 0^+} \sin\left(\frac{\pi}{2x}\right)$. (I did the first one.)

x	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	0
$\sin\left(\frac{\pi}{2x}\right)$	$\sin\left(\frac{\pi}{\frac{1}{5}}\right) = \sin(5\pi) = \boxed{0}$			

Use the table to make a guess about $\lim_{x\to 0^+} \sin\left(\frac{\pi}{2x}\right) =$

(b) Find the value of $\sin\left(\frac{\pi}{2x}\right)$ when $x = \frac{1}{1001}$. Does this change your guess about $\lim_{x\to 0^+} \sin\left(\frac{\pi}{2x}\right)$?

(c) Use your phone to graph $\sin\left(\frac{\pi}{2x}\right)$ at `www.desmos.com` or `www.wolframalpha.com`. Give your final answer to $\lim_{x\to 0^+} \sin\left(\frac{\pi}{2x}\right)$ below. Make sure to explain!

6. Find the following given that $g(x) = \begin{cases} \ln x, & \text{if } 0 < x < 1 \\ e^{x-1} - 1, & \text{if } 1 < x \le 2 \\ x + e, & \text{if } x > 2 \end{cases}$.

(a) $\lim_{x\to 1^+} g(x) =$

(b) $\lim_{x\to 1^-} g(x) =$

(c) $\lim_{x\to 1} g(x) =$

(d) $\lim_{x\to 2^+} g(x) =$

(e) $\lim_{x\to 2^-} g(x) =$

(f) $\lim_{x\to 2} g(x) =$