

07 – Limits at Infinity

Definition: Limits at Infinity (Informally)

Suppose f is defined on some interval (a, ∞) . We write $\lim_{x \rightarrow \infty} f(x) = L$ if the values of $f(x)$ can be made to stay arbitrarily close to L by taking x sufficiently large.

- We similarly define $\lim_{x \rightarrow -\infty} f(x) = L$.

1. Find the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{1}{x}$

(d) $\lim_{x \rightarrow -\infty} \left(\frac{1}{x} + 2 \right)$

(b) $\lim_{x \rightarrow \infty} e^x$

(e) $\lim_{x \rightarrow -\infty} e^x$

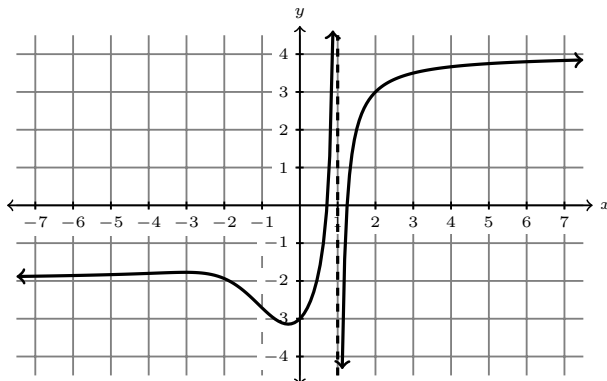
(c) $\lim_{x \rightarrow \infty} \sin(x)$

(f) $\lim_{x \rightarrow -\infty} \arctan(x)$

Definition: Horizontal Asymptote

A horizontal line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if at least one of the following are true: $\lim_{x \rightarrow \infty} f(x) = L$ or $\lim_{x \rightarrow -\infty} f(x) = L$.

2. Find all vertical and horizontal asymptotes of the graph given below.



(a) Vert. asymptotes: _____

(b) Hor. asymptotes: _____

3. Find the following limits.

(a) $\lim_{x \rightarrow -\infty} \frac{2x^3 + x + 1}{7 + 5x^3}$

(b) $\lim_{x \rightarrow -\infty} \frac{2x^3 + x + 1}{\sqrt{7 + 5x^6}}$

(c) $\lim_{x \rightarrow \infty} (e^x - xe^x)$