Author 1 _____

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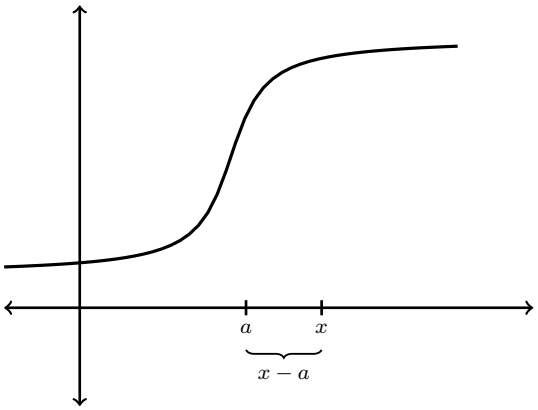
08 – Tangents & Velocity

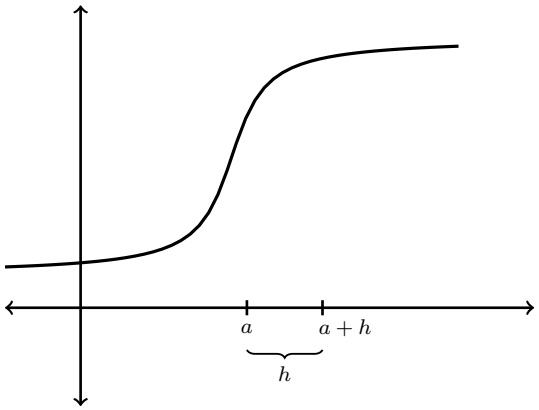
1. Suppose that the graph of $y = f(x)$ is given below. Let a be an arbitrary number.

- Label the points $(a, f(a))$ and $(x, f(x))$ on the graph on the **left**.
- Draw the line through the points $(a, f(a))$ and $(x, f(x))$ on the graph on the **left**.
- Write an equation for the slope of this line underneath it.

- Label the points $(a, f(a))$ and $(a + h, f(a + h))$ on the graph on the **right**.
- Draw the line through the points $(a, f(a))$ and $(a + h, f(a + h))$ on the graph on the **right**.
- Write an equation for the slope of this line underneath it.



Slope =



Slope =

Definition: Tangent Line

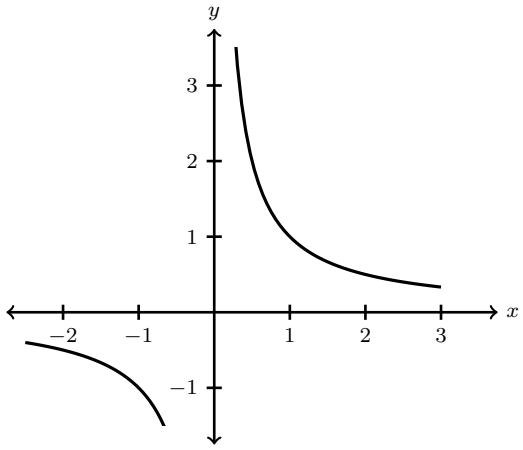
The **tangent line** to the graph of $f(x)$ at $x = a$ is the line through $(a, f(a))$ with

$$\text{slope} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

provided the limit exists. (Both limits yield the same answer, so you can choose which to use.)

2. On **both** of the graphs above (in part 1), draw the tangent line to the curve $y = f(x)$ when $x = a$.

3. The graph of the curve $y = \frac{1}{x}$ is below. Draw the tangent line to the curve $y = \frac{1}{x}$ at the point $(2, \frac{1}{2})$, **and** find an equation for the tangent line to the curve $y = \frac{1}{x}$ at the point $(2, \frac{1}{2})$.



Definition: Velocity

If $s(t)$ gives the position of an object at time t, then the (instantaneous) **velocity** of the object at time $t = a$ is

$$v(a) = \lim_{t \to a} \frac{s(t) - s(a)}{t - a} = \lim_{h \to 0} \frac{s(a + h) - s(a)}{h}$$

provided the limit exists. (Both limits yield the same answer, so you can choose which to use.)

4. Suppose a ball is dropped from a bridge, and its height in feet after t seconds is given by

$$s(t) = -16t^2 + 1600.$$

With what velocity will the ball hit the ground? *Hint: first figure out <u>when</u> the ball hits the ground.*