

# 09 – The Derivative

## Definition: The Derivative at $a$

The **derivative of  $f$  at  $a$** , is a *number* denoted  $f'(a)$ , defined by

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

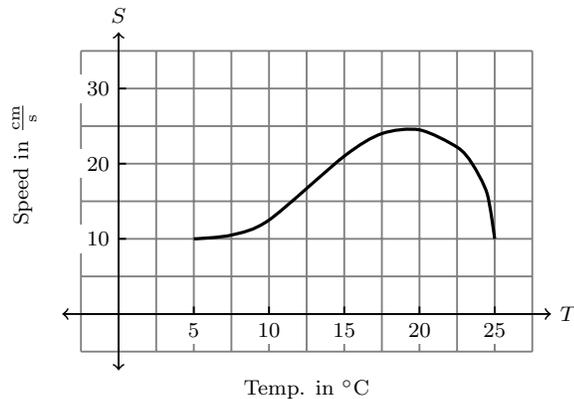
provided the limit exists. (Both limits yield the same answer, so you can choose which to use.)

## Definition: The Derivative Function

The **derivative of  $f$** , is a *function* denoted  $f'(x)$ , defined by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

1. The graph below shows the influence of the temperature  $T$  on the maximal sustainable swimming speed  $S$  of Coho salmon. Estimate  $S'(15)$  (include the units), and describe the meaning of  $S'(15)$ . *Make sure to show your work, which may include drawing on the graph.*

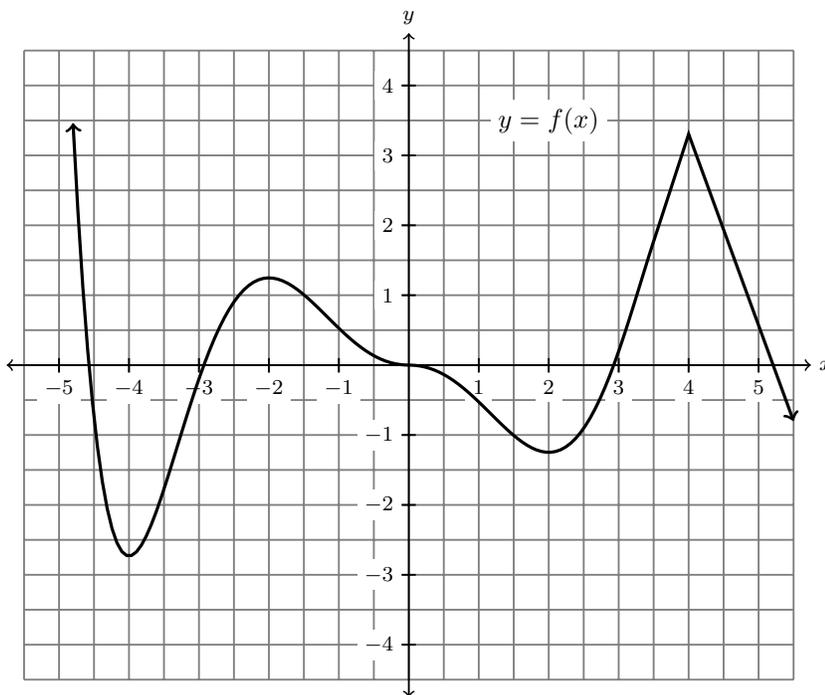


2. Let  $f(x) = x^2 - 2x$ . Find a formula for  $f'(x)$ , and use it to compute  $f'(0)$ ,  $f'(1)$ , and  $f'(2)$ .

### Definition: Differentiability

We say  $f$  is **differentiable at  $a$**  if  $f'(a)$  exists.

3. The graph of  $y = f(x)$  is given below.



- (a) Use the graph above to fill in the table below for  $f'(x)$ .  
 (b) Determine all  $x$ -values (if any) where  $f$  is not differentiable:  $x =$  \_\_\_\_\_  
 (c) Sketch the graph of  $f'(x)$  below.

| $x$ | $f'(x)$ |
|-----|---------|
| -5  |         |
| -4  |         |
| -3  |         |
| -2  |         |
| -1  |         |
| 0   |         |
| 1   |         |
| 2   |         |
| 3   |         |
| 4   |         |
| 5   |         |

