

Modern Algebra
MATH 325W – Spring 2015

Monday: Chapter 18: Subgroups
Wednesday: Chapter 19: Generating Subgroups
Friday: Chapter 20: Cosets

Week 12

HOMEWORK

Homework #20

due Tuesday, April 21

Ch. 17: #17, 18, 22, 23

AP #1: Determine whether the given function is (1) one-to-one, (2) onto, and (3) a homomorphism. In each case, either give a proof or a counterexample.

$$f : (\mathrm{GL}_2(\mathbb{R}), \cdot) \rightarrow (\mathbb{R}^\times, \cdot), \quad f(A) = \det A$$

Recall that we defined $\mathrm{GL}_2(\mathbb{R})$ to be the set of all 2×2 matrices with nonzero determinant, and we discussed why this is a group with respect to matrix multiplication. You may freely use theorems from linear algebra.

Homework #21

due Friday, April 24

Ch. 18: #4, 8, 10, 12

Ch. 19: #8, 14, 20, 21

WRITING ASSIGNMENTS

On writing assignments, part of your grade will reflect the quality of your *style*. Style includes everything from the basic mechanics of writing (complete, grammatically correct sentences with capitalization and proper punctuation) to the conventions of writing mathematics developed in Linear Algebra.

Writing Assignment #10

due Wednesday, April 22

Ch. 16: #35

Hint: define $\alpha = \sigma\tau\sigma^{-1}$. You want to show that the number of elements of $\{1, \dots, n\}$ left fixed by τ is the same as that for α , i.e. that $\mathrm{Ch}(\tau) = \mathrm{Ch}(\alpha)$. First show that if τ leaves x fixed then α leaves $\sigma(x)$ fixed, i.e. show that $\tau(x) = x$ implies that $\alpha(\sigma(x)) = \sigma(x)$. Explain, carefully, why this implies that $\mathrm{Ch}(\tau) \leq \mathrm{Ch}(\alpha)$. Now assume that α leaves y fixed. What does this imply that τ must fix? Use this to show that $\mathrm{Ch}(\tau) \geq \mathrm{Ch}(\alpha)$.

Ch. 18: #32

AP #1: Let G be group, and let $f : G \rightarrow G$ be the function defined by $f(g) = g^{-1}$. Prove that G is abelian **if and only if** f is a homomorphism.