

Modern Algebra
MATH 325W – Spring 2015

Monday: Chapter 7: Solvability by Radicals
Wednesday: Chapter 7: Rings
Friday: Review for Exam 1

Week 5

HOMEWORK

Homework #8

due Tuesday, February 17

Ch. 6: 2, 3, 4, 5, 14, 18

For 14 and 18, you can freely use that $\alpha_1, \alpha_2 \in \mathbb{Q}(\beta_1, \beta_2) \implies \mathbb{Q}(\alpha_1, \alpha_2) \subseteq \mathbb{Q}(\beta_1, \beta_2)$.

Homework #9

due Friday, February 20

Ch. 6: 25, 29

Hint: problem 53 from Chapter 3 may be helpful.

Ch. 7: Do the following three part question.

Additional Problem #1. Let $s(x) = x^2 + 3x + 2$, $t(x) = x^2 + 4x + 5$, and $p(x) = x + 2$.

- (a) Does there exist $q(x) \in \mathbb{Q}[x]$ such that $s(x) = q(x)p(x)$, i.e. is $s(x)$ divisible by $p(x)$? If so, what is $q(x)$? If not, why not?
- (b) Does there exist $q(x) \in \mathbb{Q}[x]$ such that $t(x) = q(x)p(x)$, i.e. is $t(x)$ divisible by $p(x)$? If so, what is $q(x)$? If not, why not? *Hint: not.*
- (c) Find $q(x), r(x) \in \mathbb{Q}[x]$ with $\deg(r(x)) < \deg(p(x))$ such that $t(x) = q(x)p(x) + r(x)$.

WRITING ASSIGNMENTS

On writing assignments, part of your grade will reflect the quality of your *style*. Style includes everything from the basic mechanics of writing (complete, grammatically correct sentences with capitalization and proper punctuation) to the conventions of writing mathematics developed in Linear Algebra.

Writing Assignment #4

due Wednesday, February 18

Ch. 5: 12, 13

Ch. 6: 33

There is a typo in 33. The problem should read: "... then $\mathbb{Q}^{p(x)} \neq \mathbb{Q}(r_1, r_2)$."

Extra Credit: Let $\alpha = \sqrt{2} - i$. Give a description of $\mathbb{Q}(\alpha)$ analogous to $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} | a, b \in \mathbb{Q}\}$. Make sure to prove everything. *Please do not use outside resources, e.g. no rhuben. Only correct, well-justified, and beautifully written proofs will receive points!*