An Introduction to the Theory of Groups

a.k.a. Senior Seminar in Algebra

MATH 437.01 – Fall 2014

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Office Hours 2:30PM–4:30PM MWF or by appointment.

Course Webpage http://people.hamilton.edu/jwiscons/teaching/math437_f14.html

Course Overview The course will be structured to build both intuition and theory with the generation and exploration of student-generated conjectures being the glue. The main theoretical objective is to develop enough machinery to prove Sylow's theorems from finite group theory; however, this objective will be reached early enough for you to conclude the course with a research project of your own choosing. Possible topics for the project include: the countable random graph and its automorphism group, symplectic spaces and symplectic groups, groups associated to games such as Spinpossible™ (https://spinpossible.com), reflection groups and Coxeter groups, free groups, or the classification of the finite multiply transitive groups.

Course Components

Daily Homework. Homework will typically be assigned each class meeting, due at the next meeting. This will usually consist of solving problems and proving theorems from the class notes. You are *allowed and encouraged* to work together on homework, but you are expected to **write up your solutions on your own**. The majority of class time will be occupied with the class presenting problems from the daily homework, and during this time you are encouraged to annotate or rewrite your own solutions with a pen provided by me in class. And this is important: **I do not want you using outside resources for this component of the course; I do want you to create mathematics on your own and in collaboration with your classmates.**

Weekly Homework. Each week you will be responsible for revising and typing up, in LaTeX, two starred problems from the previous week. I will support you as much as needed with LaTeX, and for those of you unfamiliar with LaTeX, ShareLaTeX is a good place to start.

Participation. A typical class meeting will consist of students presenting solutions to problems and proofs of theorems from the class notes. To receive a passing grade for this component, **you must present at least once every two weeks**. The goals of the presentations are (1) for the speaker to effectively and clearly communicate their solution to the class and (2) for the class to meaningfully engage the speaker with comments and questions meant to clarify, challenge, and quite possibly correct the argument being presented. You will be graded on both aspects.

Follow-up Questions. Class will often begin with a handful of *basic* questions on *previously covered* material being asked to randomly chosen students.

Project. Near the end of the course, you will be responsible for researching a group-theoretical topic of your own choosing. You do not need to prove something original, but you will be required to write up an introduction to your topic, including motivation, definitions, statements of theorems, and proofs (in your own words!) of nontrivial results. You will also give a 20-25 minute in-class presentation on your topic. The goal of this component is simply to keep the following words of Paul Halmos in mind.

> *Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis?*

Grade Composition

Daily Homework	25%
Weekly Homework	25%
Participation	20%
Follow-up questions	5%
Project	25%

Grading Rubrics and language are borrowed from one of my teaching heroes, Dana Ernst.

Daily Homework. ✓-system

Weekly Homework. $4-0$ scale:

4	This is correct and well-written mathematics!
3	This is a good piece of work, yet there are some mathematical errors or some writing errors that need addressing.
2	There is some good intuition here, but there is at least one serious flaw.
1	I dont understand this, but I see that you have worked on it; come see me!
0	I believe that you have not worked on this problem enough or you didnt submit any work.

Presentations. $4-0$ scale:

4	Completely correct and clear proof or solution!
3	Proof has minor technical flaws, some unclear language, or lacking some details. Essentially correct.
2	A partial explanation or proof is provided but a significant gap still exists to reach a full solution or proof.
1	Minimal progress has been made that includes relevant information and could lead to a proof or solution.
0	You were completely unprepared.

Follow-up Questions. $4-0$ scale:

4	Completely correct and clear response!
3	Response has minor technical flaws, some unclear language, or lacking some details. Essentially correct.
2	A partial explanation is provided but a significant gap still exists.
0	A response is given that does not address the question.

Getting Help Mathematics is hard. Try hard. But don't be surprised if that is not always enough. Talk with your classmates. Talk with me. But please try to avoid asking "how do I start." Instead, try to rewrite the problem in a way that is more meaningful to you and then ask, "does my interpretation of the question seem correct." Very often, the act of "simply" reformulating a problem will lead to insight about its answer.

Disabilities Any student with a documented disability needing academic adjustments or accommodations should speak with me during the first two weeks of class. All discussions will remain confidential. Students with disabilities should contact Allen Harrison in the Dean of Students Office (Elihu Root House; ext. 4021).