

Math 1300 Fall 2005
Review Sheet for Midterm Exam 2

1. (a) Complete the following definition: A function $f(x)$ is continuous at $x = a$ if:
(b) Use part (a) to explain why a polynomial is continuous everywhere.
(c) Find the values of x (if any), at which f is not continuous, and determine whether each such value is a

removable discontinuity: $f(x) = \begin{cases} 2x - 3, & x \leq 2, \\ x^2, & x > 2; \end{cases}$

2. Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}$.
(b) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x}$.
(c) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x \sin x}$.
(d) $\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$.

3. Use the Intermediate Value Theorem to prove that the equation $x^3 = 3$ has a solution in the interval $[0, 2]$.

4. While sightseeing at the Great Crater Canyon on Planet Mathenola, Newtus Calculori "accidentally" dropped his calculus book off the side and watched in shock as it fell straight to the bottom of the 162 meter canyon. Being a great calculus student, Newtus was able to determine that the book fell at such a rate that t seconds after the book fell, it had dropped a distance of $s = 6t^3$ m.

- (a) How many seconds after Newtus dropped the book did it hit the bottom of the canyon?
(b) What was the average velocity of the book during the first 2 seconds of its descent?
(c) What was the instantaneous speed of the book when it hit the bottom of the canyon?

5. An automobile, starting at rest, accelerates steadily down a straight road until time $t = 10$ s, after which it travels at a constant velocity until time $t = 20$ s.

- (a) Draw a possible graph of velocity vs. time.
(b) Draw a possible graph of position vs. time.

6. (a) Find $f'(x)$, USING THE DEFINITION OF DERIVATIVE, if $f(x) = \sqrt{x+1}$.

- (b) Find $f'(x)$, USING THE DEFINITION OF DERIVATIVE, if $f(x) = \frac{1}{x^2}$.

7. Find $f'(x)$ if

(a) $f(x) = (x^2 + 4)(x^2 - 4)$

(b) $f(x) = \frac{x + x^2}{x^3}$

(c) $f(x) = 4x^{-3} + 2x^{-2} + x$

(d) $f(x) = \frac{1 - \cos^2 x}{\sin x}$

(e) $f(x) = \sec^2 x - \tan^2 x$

(f) $f(x) = (1 + \sin^3(x^5))^{12}$

8. (a) If $f(x) = (x^2 + x)(x^2 - x)$, find $f'(x)$ and the equation of the tangent line to the graph of $y = f(x)$ at $x = 1$.

- (b) Find all values of x in the interval $[-2\pi, 2\pi]$ at which the graph of the function $f(x) = x + \sin x$ has a horizontal tangent line.

9. Harold and Kumar both leave Whit Castle at the same time. Harold walks due North at 1.5 miles per hour while Kumar walks due East at 2 miles per hour. How fast is the distance between them changing after 2 hours?