

1. (a) A function, $f(x)$, is continuous at $x = a$ if: $f(a)$ is defined, $\lim_{x \rightarrow a} f(x)$ exists, and $\lim_{x \rightarrow a} f(x) = f(a)$.

(b) Polynomials are defined everywhere, and if $p(x)$ is a polynomial, Theorem 2.2.3 tells us that

$$\lim_{x \rightarrow a} p(x) = p(a) \text{ for any } a.$$

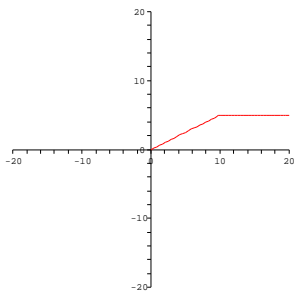
(c) $f(x)$ has a non-removable discontinuity at $x = 2$ and is continuous everywhere else.

2. (a) 0 (b) 0 (c) 1 (d) 2

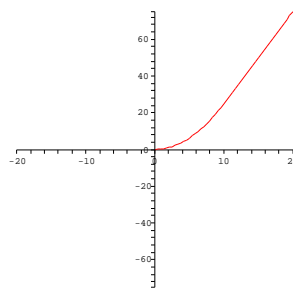
3. Let $g(x) = x^3 - 3$. Then, $g(0) = -3$ and $g(2) = 5$, so $g(0) < 0 < g(2)$. Since, $g(x)$ is continuous on $[0, 2]$ (because polynomials are continuous everywhere), we can apply the IVT, which tells us that there is a k between 0 and 2 such that $g(k) = 0$. Therefore, $k^3 - 3 = 0$, so $k^3 = 3$. Thus, k is a solution to the equation $x^3 = 3$, and k is in $[0, 2]$.

4. (a) 3 sec (b) 24 m/sec (c) 162 m/sec

5.



Velocity vs. Time



Position vs. Time

6. (a) $f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} = \dots = \frac{1}{2\sqrt{x+1}}$ (b) $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \dots = -\frac{2}{x^3}$

7. (a) $4x^3$ (b) $\frac{x^3(1+2x) - (x+x^2)(3x^2)}{x^6} = -\frac{2}{x^3} - \frac{1}{x^2}$ (c) $-12x^{-4} - 4x^{-3} + 1$

(d) $\frac{\sin x(-2 \cos x(-\sin x)) - (1 - \cos^2 x)(\cos x)}{\sin^2 x} = \cos x$ (NOTE: $1 - \cos^2 x = \sin^2 x$) (e) $2 \sec x \sec x \tan x - 2 \tan x \sec^2 x = 0$ (NOTE: $\sec^2 x - \tan^2 x = 1$) (f) $12(1 + \sin^3(x^5))^{11}(3 \sin^2(x^5))(\cos(x^5))(5x^4)$

8. (a) $f'(x) = 4x^3 - 2x$, $f'(1) = 2$, and the equation of the line tangent to the graph of f at $x = 1$ is $y - 0 = 2(x - 1)$. (b) $-\pi$ and π

9. 2.5 miles/hr