Math 1300 Spring 2006
Review Sheet Answer Key for Midterm Exam 2

The solutions were typed up fast and very well may contain errors. Hope this helps.

1. (3.2) Use the **definition of the derivative** to find $\frac{dy}{dx}$ if $y = \frac{1}{x+1}$.
ans. $\frac{dy}{dx} = \lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \cdots = \frac{-1}{(x+1)^2}$

2. (3.2) Let $y = \frac{1}{\sqrt{x}}$. Find the derivative of y using
(i) the limit definition of the derivative
ans. $\frac{dy}{dx} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \cdots = -\frac{1}{2x^{\frac{3}{2}}}$

(ii) the power rule
ans. $\frac{dy}{dx} = \frac{d}{dx}[x^{-\frac{1}{2}}] = \cdots = -\frac{1}{2}x^{-\frac{3}{2}}$

3. Find $\frac{dy}{dx}$.

1. (3.3) $y = e^2$; $\frac{dy}{dx} = 0$
2. (3.5) $y = \frac{2\sin(x)\cos(x)}{\cos(2x)}$; $\frac{dy}{dx} = 2\sec^2(2x)$
3. (3.6) $y = \sin^2(\sin^2(x))$; $\frac{dy}{dx} = 2\sin(\sin^2 x)\cos(\sin^2 x)2\sin x\cos x$
4. (4.1) $\sin(y) = \ln(x+y)$; $\frac{dy}{dx} = \frac{\frac{1}{x+y}}{\cos y - \frac{1}{x+y}}$
5. (4.2) $y = \ln(\cos x)$; $\frac{dy}{dx} = -\frac{\sin x}{\cos x}$
6. (4.2) $y = \ln\left(\frac{(x^2+3)^3\sqrt{\sin x - 2}}{(2x+7)^4}\right)$; $\frac{dy}{dx} = \frac{6x}{x^2+3} + \frac{\frac{1}{2}\cos x}{\sin x - 2} - \frac{\frac{1}{2}}{2x+7}$
7. (4.3) $y = \frac{3}{\sec^{-1}(2x^2)}$; $\frac{dy}{dx} = \frac{\frac{-3}{2x^2\sqrt{4x^4-1}}}{(\sec^{-1}(2x^2))^2}$
8. (4.3) $y = e^{\sin(x)}$; $\frac{dy}{dx} = e^{\sin(x)}\cos(x)$

4. (3.5) Find $f'(x)$ and $f'\left(\frac{\pi}{3}\right)$ if $f(x) = \sin(x)\cos(x)$
$f'(x) = -\sin^2 x + \cos^2 x$, $f'\left(\frac{\pi}{3}\right) = -\frac{1}{2}$

5. (4.1) Find all values of x at which the curve $y^3 + yx^2 + x^2 - 3y^2 = 0$ has a horizontal tangent line.
ans. $(0,0), (0,3)$

6. (3.3) Find an equation for the tangent line to the graph of $y = (5x^2 - 3)(7x^3 + x)$ at $x = 4$.
ans. $y - 34804 = 44029(x - 4)$.

7. (4.4) Calculate $\lim_{x \to \infty}(1 + \frac{1}{x})^{-x}$.
ans. e^{-x}

8.

1. (3.7) Erika is cleaning the gutters on her house using a 10ft ladder propped up against the wall. Emily pulls the base of the ladder away from the wall at a rate of 3 ft/sec. how fast is the top of the ladder falling down the wall when it is 6 ft from the ground? (possible hint: your answer should be negative)
-4 ft/sec.

2. (3.7) The public health spending (in dollars) of a certain town is given by the equation $S = 7500 \cdot \ln(2p + 5000)$, where p is the population of the town. If the population of the town is growing at a steady rate of 100 people per year, at what rate is the public health spending of the town increasing when the population is 5000 people?
100 dollars/year

3. (4.1) The minute hand of a watch begins melting at a certian rate when it reaches the top of the hour. As the hand begins moving, what is the formula for the rate at which the area swept out by the hand is changing? (hint: The area for the whole circle is πr^2 - find a relation between any generic angle θ and the rotation around a whole circle to find the area of a portion of a circle in terms of r and θ)
$A = \frac{\theta r^2}{2}$, so $\frac{dA}{dt} = \frac{r^2}{2}\frac{d\theta}{dt}$