Math 1300 Fall 2005
Review Sheet for Midterm Exam 3

1. Use linear approximation to find $\sin(47°)$.

2. Use an appropriate local linear approximation to estimate $\sqrt{48}$.

3. Assume the earth is a perfect sphere, and that the radius of the earth at the equator is 3960 miles, or 20,908,800 feet. Imagine that a string is wrapped tightly around the earth at the equator. Then imagine that the string is lengthened by an amount that allows it to be strung all the way around the earth at the equator on short poles that are $\frac{1}{\pi}$ feet above the ground. ($\frac{1}{\pi}$ feet is a little less than $\frac{1}{3}$ of a foot, or about 4 inches). Use differentials to approximate the increase in the length of the string.

4. Find $\frac{dy}{dx}$ for the following functions (some of which may be defined implicitly):
(a) $y = \ln(\sin(x) + x^3)$
(b) $y = e^{(\sin(x)+x^3)}$
(c) $y = \tan^{-1}(\ln x)$
(d) $y = \cos(\ln x)$
(e) $y = \ln(\tan^{-1}(x^2))$
(f) $y = x^{\sin x}$
(g) $ye^x = xe^y$

5. Evaluate the following limits:
(a) $\lim_{x\to+\infty} \frac{2x^6+5x^4+7x}{7x^6-3x^5-4x^3+10x}$
(b) $\lim_{x\to+\infty} x^{1/x}$
(c) $\lim_{x\to+\infty} \frac{1}{x} - \frac{1}{e^x-1}$
(d) $\lim_{x\to+\infty} x^{1/x}$
(e) $\lim_{x\to+\infty} x \ln\left(\frac{x-1}{x+1}\right)$
(f) $\lim_{x\to+\infty} \left(\frac{1}{x}\right)(\ln x)$
(g) $\lim_{x\to+\infty}(x + \sin x)^x$
(h) $\lim_{x\to 0} \frac{(1-\cos x)}{(xe^{2x}-x)}$

6. Given

$$f(x) = \frac{x^3}{x^2+1}; \qquad f'(x) = \frac{x^2(x^2+3)}{(x^2+1)^2}; \qquad f''(x) = \frac{2x(3-x^2)}{(x^2+1)^3};$$

find:
(a) x and y intercepts
(b) vertical and horizontal asymptotoes
(c) critical points - classify each as a relative maximum, relative minimum, or neither
(d) intervals where f is increasing and decreasing
(e) inflection points
(f) intervals where f is concave up and concave down
(g) Sketch the curve

7. Same as #6, given

$$f(x) = x^4 - 4x^2 - 1, \qquad f'(x) = 4x^3 - 8x, \qquad f''(x) = 12x^2 - 8.$$

8. What is the length of the shortest line segment lying wholly in the first quadrant tangent to the graph of $y = \frac{1}{x}$ and with its endpoints on the coordinate axes?

9. A closed cylindrical can is made using 2 sq. ft of material for the sides, top and bottom. What height and radius would maximize the volume of the can?

10. Let $f(x) = x^5 + 2x + 1$.

1. Use the Intermediate Value Theorem to show that the equation $f(x) = 0$ has *at least one* solution on the interval [-1,0].
2. Use the Mean Value Theorem to show that there is *exactly one* solution to the equation $f(x) = 0$ on the interval [-1,0].

11. Let $f(x) = x^2$ on the interval [a,b] for $a < b$. Which of the following values of c satisfy the conclusion of the Mean Value Theorem for f?

1. $c = \dfrac{a^2 + b^2}{2}$
2. $c = b^2 - a^2$
3. $c = \dfrac{a + b}{2}$
4. $c = b - a$

12. Use the Mean Value Theorem to prove Rolle's Theorem.

13. Suppose Harold and Kumar race each other to White Castle and arrive at the same time. Use the MVT to show that there was at least one moment when they had the same velocity.