MATH 1300: CALCULUS 1

May 9, 2006

Final Exam

YOUR NAME:

- ◯ **001** E. Mankin .... (8am)
- ◯ **002** N. Flores .... (9am)
- ◯ **003** R. Chestnut .... (9am)
- ◯ **004** E. Frugoni .... (10am)
- ◯ **005** J. Fuhrmann .... (11am)
- ◯ **006** J. Sanders .... (11am)
- ◯ **007** J. Nibert .... (12pm)
- ◯ **008** M. Walter .... (2pm)
- ◯ **009** T. Schumacher .... (2pm)
- ◯ **011** J. Wiscons .... (10am)
- ◯ **012** V. Wong .... (12pm)
- ◯ **013** S. Tramer .... (1pm)
- ◯ **014** C. Moody .... (1pm)
- ◯ **015** J. Johanson .... (3pm)

Show all your work.
Answers out of the blue and without any supporting work will receive no credit *even if they are right!*
(Does not apply to multiple choice and true/false questions)

Write clearly. **Box your final answers.**
No calculators allowed. **No cheat sheets allowed.**

DO NOT WRITE ON THIS BOX!

problem	points	score
1	24 pts	
2	8 pts	
3	8 pts	
4	9 pts	
5	8 pts	
6	8 pts	
7	6 pts	
8	7 pts	
9-26	72 pts	
TOTAL	150 pts	

1: *(24 points)* Evaluate the following definite and indefinite integrals:

(a) $\displaystyle\int_0^1 (2x-1)^{42}\,dx$

(b) $\displaystyle\int_{\ln 2}^{\ln 3} e^{2x}\,dx$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x \cos x\,dx$

(d) $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

(e) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

(f) $\displaystyle\int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$

2: *(8 points)* Find the area under the curve $y = e^x + x$ over the interval $[0, 1]$.

3: *(8 points)* Determine $f'(x)$ if $f(x) = \displaystyle\int_2^x \sin(t^2)\,dt$.

4: *(9 points)* Given that $\int_0^{2\pi} f(x)\ dx = 3$ and $\int_0^{2\pi} g(x)\ dx = -\frac{1}{2}$, determine the following quantities:

(a) $\int_{2\pi}^{0} f(x)\ dx$

(b) $\int_0^{2\pi} (2g(x) - 5f(x))\ dx$

(c) $\int_0^{2\pi} (3 - g(x))\ dx$

5: *(8 points)* Find the area of the region enclosed by the curves $y = x^2 - x - 4$ and $y = x - 1$.

6: *(8 points)* Set up, but DO NOT EVALUATE, integrals that express the volume of the solids that result when the region enclosed by the curves

$$y = e^x, \quad x = 0, \quad x = \ln(2), \quad y = 0$$

is revolved about:

(a) the x-axis

(b) the y-axis

7: *(6 points)* Give an example of a function $f(x)$ that is continuous everywhere, but is not differentiable everywhere. At what value(s) of x is $f(x)$ *not* differentiable?

8: *(7 points)* If $f(x) = x^4 - 3x^3 + 1$, prove that there is a point on the interval $[0, 2]$ where $f'(c) = -4$.

Proof (fill in the blanks and note that **there are 7 blanks**):

$f(x)$ is continuous on $[0, 2]$ because f is a ________________________ .

$f'(x) = 4x^3 - 9x^2$, so f is ________________________ on $(0, 2)$.

So, by the ________________________ theorem, there is a point c in $(0, 2)$ such that

$$f'(c) = \frac{\_\_\_\_ - \_\_\_\_}{\_\_\_\_ - \_\_\_\_} = \frac{-7-1}{2} = -4$$

The remainder of this exam consists of **True/False** and **Multiple Choice** questions *(worth 4 points each)*. Please circle your answer for each question on **THIS** page. (The actual questions are on the pages that follow.) **It is not necessary to show your work, and no partial credit will be given.**

9: **(TRUE)** **(FALSE)**

10: **(TRUE)** **(FALSE)**

11: **(TRUE)** **(FALSE)**

12: **(TRUE)** **(FALSE)**

13: **(A)** **(B)** **(C)** **(D)** **(E)**

14: **(A)** **(B)** **(C)** **(D)** **(E)**

15: **(A)** **(B)** **(C)** **(D)** **(E)**

16: **(A)** **(B)** **(C)** **(D)** **(E)**

17: **(A)** **(B)** **(C)** **(D)** **(E)**

18: **(A)** **(B)** **(C)** **(D)** **(E)**

19: **(A)** **(B)** **(C)** **(D)** **(E)**

20: **(A)** **(B)** **(C)** **(D)** **(E)**

21: **(A)** **(B)** **(C)** **(D)** **(E)**

22: **(A)** **(B)** **(C)** **(D)** **(E)**

23: **(A)** **(B)** **(C)** **(D)** **(E)**

24: **(A)** **(B)** **(C)** **(D)** **(E)**

25: **(A)** **(B)** **(C)** **(D)** **(E)**

26: **(A)** **(B)** **(C)** **(D)** **(E)**

9: TRUE or FALSE: Two different functions can have the same derivative.

10: TRUE or FALSE: Two different functions can have the same antiderivative.

11: TRUE or FALSE: An x-coordinate where $f''(x) = 0$ is a point of inflection.

12: TRUE or FALSE: $\int xe^x dx = xe^x - e^x + C$.

13: $\lim_{x\to\infty} \dfrac{4x^{10} + 3x}{5x^{10} + e^x} =$

(A) ∞ **(B)** $\frac{4}{5}$ **(C)** 0 **(D)** $-\infty$ **(E)** DNE

14: Find the x-coordinates of the absolute maximum and absolute minimum of $f(x) = \dfrac{1}{3}x^3 - \dfrac{1}{2}x^2 - 6x$ on $[-1, 4]$.

(A) Max: $x = -2$ Min: $x = 3$
(B) Max: $x = 4$ Min: $x = 3$
(C) Max: $x = 3$ Min: $x = -1$
(D) Max: $x = 4$ Min: $x = -1$
(E) None of the above

15: Find $F'(1)$ if $F(x) = e^2$.

(A) e^2 **(B)** 1 **(C)** $2e$ **(D)** $\frac{1}{e}$ **(E)** 0

16: If $a < 0$ and $b > 0$, then $\lim_{x\to\infty} \dfrac{ax^4 + bx + c}{bx^3 + c} =$

(A) 0 **(B)** $-\infty$ **(C)** ∞ **(D)** $\frac{a}{b}$ **(E)** 1

17: Which of the following is true about $f(x) = \dfrac{x^2 + 4x + 4}{x^2 - x - 6}$?

(A) f has a vertical asymptote at $x = -2$
(B) f has a horizontal asymptote of $y = 1$
(C) f has no vertical asymptote
(D) f has no horizontal asymptote
(E) Both (A) and (B)

18: Find $\dfrac{dy}{dx}$ by implicit differentiation if $y = x^2y^2 - 3xy - 10$

(A) $2x^2y - 3x$ **(B)** $\dfrac{10}{4xy - 3x - 1}$ **(C)** $\dfrac{-2xy^2 + 3y + 1}{2x^2y + 3x}$ **(D)** $2xy^2 - 3y$ **(E)** $\dfrac{-2xy^2 + 3y}{2x^2y - 3x - 1}$

19: If $g(1) = 2$, $f(1) = -1$, $g'(1) = 0$, and $f'(1) = 7$, then what is $(f/g)'(1)$?

(A) 0 **(B)** $\frac{7}{2}$ **(C)** $-\frac{1}{2}$ **(D)** -7 **(E)** DNE

20: Find an equation for the tangent line to the curve $y = x^7 - 5$ at the point $(1, -4)$.

(A) $y = 7x$ **(B)** $y = 7x + 5$ **(C)** $y = 7x - 3$ **(D)** $y = 7x - 11$ **(E)** $y = 7x - 6$

21: Find $\frac{d}{dx}\left[\cos^2(\pi x)\right]$.

(A) $\sin^2(\pi x)$ **(B)** $-2\pi\cos(\pi x)\sin(\pi x)$ **(C)** $2\pi\cos(\pi x)\sin(\pi x)$
(D) $2\cos(\pi x)$ **(E)** None of the above

22: Find $\lim_{x\to 3}\left(\frac{1}{x-3}\right)$.

(A) ∞ **(B)** $-\infty$ **(C)** $-\frac{1}{3}$ **(D)** 0 **(E)** DNE

23: Evaluate $\int \left(\csc^2\theta - \sec^2\theta\right) d\theta$.

(A) $-\cot\theta - \tan\theta + C$ **(B)** $\cot\theta - \tan\theta + C$ **(C)** $\cot\theta + \tan\theta + C$
(D) $-\cot\theta + \tan\theta + C$ **(E)** None of the above

24: Which of the following functions is not continuous at $x = 1$?

(A) $f(x) = \sin\left(\frac{1}{x} - x\right)$

(B) $f(x) = \sqrt{\frac{1}{2} - x}$

(C) $f(x) = |x - 1|$

(D) $f(x) = \begin{cases} \sin(\pi x), & \text{if } x \geq 1 \\ \cos(\pi x) + x, & \text{if } x < 1 \end{cases}$

(E) None of the above

25: $\lim_{h \to 0} \left(\frac{\ln(2+h) - \ln(2)}{h} \right) =$

(A) 0 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** ∞ **(E)** DNE

26: $1^3 - 1^2 + 1 + \frac{1}{2} > 0$ and $(-1)^3 - (-1)^2 + (-1) + \frac{1}{2} < 0$, so the polynomial $x^3 - x^2 + x + \frac{1}{2}$ has a root between $x = 1$ and $x = -1$ by

(A) Rolle's theorem
(B) L'Hôpital's Rule
(C) The Mean Value Theorem
(D) The Intermediate Value Theorem
(E) The Extreme Value Theorem