Math 1300 Fall 2005
Review Sheet for Final Exam

1. Evaluate the following definite and indefinite integrals:

(a) $\displaystyle\int_1^2 \frac{3x^5+7x^2+x}{x}\,dx$

(b) $\displaystyle\int \tan^2 x\,dx$

(c) $\displaystyle\int [\tfrac{3}{2}\sqrt{t}+4]\,dt$

(d) $\displaystyle\int_1^2 \left(\sqrt[3]{x^2}+\frac{4}{\sqrt[4]{x^5}}\right)dx$

(e) $\displaystyle\int x^4(x^5-2)^{3/2}\,dx$

(f) $\displaystyle\int_0^{\pi/4} \tan x\,dx$

(g) $\displaystyle\int x^5(1-x^2)^5\,dx$

(h) $\displaystyle\int_e^{e^2} \frac{(\ln x)^2}{x}\,dx$

(i) $\displaystyle\int_0^2 2xe^{x^2}\,dx$

(j) $\displaystyle\int (4x^3+1)\cos(x^4+x)\,dx$

(k) $\displaystyle\int_{-1}^4 \frac{x}{\sqrt{5+x}}\,dx$

2. Solve the following initial value problems:

(a) $\dfrac{dy}{dx}=\sqrt{x}, \qquad y(4)=0$

(b) $\dfrac{dy}{dx}=3x^2+\dfrac{2}{x^2}, \qquad y(1)=2$

3. Find the exact area under the curve $f(x)=x-1$ over the interval $[1,3]$, using Riemann sums with right-hand endpoints (i.e., $x_k^*=x_k=a+k\Delta x$). You may find the follwing formulae useful:

$$\sum_{k=1}^{n} 1 = n, \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

4. Evaluate the integral $\displaystyle\int_{-2}^{2} f(x)\,dx$, given that

$$f(x)=\begin{cases} x^2 & x>0 \\ x & x\le 0 \end{cases}$$

5. Define $F(x)$ by

$$F(x)=\int_0^x e^t\,dt.$$

(a) Use Part 2 of the Fundamental Theorem of Calculus to find $F'(x)$.
(b) Check the result in part (a) by first integrating and then differentiating.

6. Find the area of the region enclosed by the curves $y=x^2$ and $y=\sqrt{x}$.

7. Find the area of the region enclosed by the curves $y=x$, $y=4x$, and $y=2-x$.

8. Find the volume of the solids that result when the region enclosed by the curves

$$y = 0, \quad y = x^2, \quad x = 0, \quad x = 1$$

is revolved about:
(a) the x-axis (b) the y-axis

9. Set up, but DO NOT EVALUATE, integrals that express the volume of the solids that result when the region enclosed by the curves

$$y = 1, \quad y = e^x, \quad x = 0, \quad x = 2$$

is revolved about:
(a) the x-axis (b) the y-axis

Multiple choice and **True/False** questions:

10. Find $\dfrac{dy}{dx}$ if $x^3y^4 = x^7$.

(A) $\dfrac{7x^6 - 3x^2y^4}{4x^3y^3}$ **(B)** $\dfrac{7x^6 + 3x^2y^4}{4x^3y^3}$ **(C)** $7x^6 + 3x^2y^4$ **(D)** $7x^6 - 3x^2y^4$ **(E)** 0

11. $\lim\limits_{x \to +\infty} \dfrac{e^x + 3x}{x^3} =$

(A) 0 **(B)** $+\infty$ **(C)** $-\infty$ **(D)** 1 **(E)** -1

12. The function $f(x) = -x^4 - 6x^2$ is concave up on

(A) $(-\infty, +\infty)$ **(B)** $(-\infty, -81)$ **(C)** $(-\infty, -9)$ **(D)** Nowhere **(E)** $(-9, 9)$

13. The function $f(x) = 3\sin(x^2)$ has an absolute minimum of

(A) -5 **(B)** -3 **(C)** 0 **(D)** $-\dfrac{2}{3}$ **(E)** $-\dfrac{1}{3}$

14. Express the number 60 as the sum of two nonnegative numbers whose product is as large as possible.

(A) $5, 55$ **(B)** $10, 50$ **(C)** $30, 30$ **(D)** $1, 59$ **(E)** None of the above

15. True or false: Given $f(x) = x^2 - 9$ on $[-3, 3]$, the value c that satisfies the conclusion of Rolle's Theorem is $c = 0$.

(TRUE) **(FALSE)**

16. True or false: Given $f(x) = x^3$ on $[0, 2]$, the value c that satisfies the conclusion of the Mean Value Theorem is $c = 1$.

(TRUE) **(FALSE)**