Modeling the Sea Floor Using Tension Spline Curve Radial Basis Functions

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Abstract

A map of the seafloor off the coast of California is constructed by interpolating bathymetric data with radial basis functions under tension (RBFTs). Open source tools were used to calculate and render the sea floor images. Data used in this study were acquired, processed, archived, and distributed by the Seafloor Mapping Lab of California State University Monterey Bay.

Motivation

Traditional interpolation surface construction techniques have significant limitations:

- **Domain meshing is expensive.** Even with massively parallel multi-core systems, the computing effort to generate a mesh for an unstructured domain may require hours to days of processing time [2].
- **Regularly spaced data is often assumed.** Scattered data may lead to ill-posed problems or large oscillatory artifacts.
- **They suffer from the "Curse of Dimensionality".** Higher dimension problems exhibit significantly more complex features.

RBFTs circumvent these restrictions as they are mathematically simple and truly meshless.

Multidimensional Interpolation

Denote the data points as $\{(\mathbf{x}_i, f_i)\}_{i=1}^{N}$ and τ the thin plate spline tension parameter. The interpolating surface function is

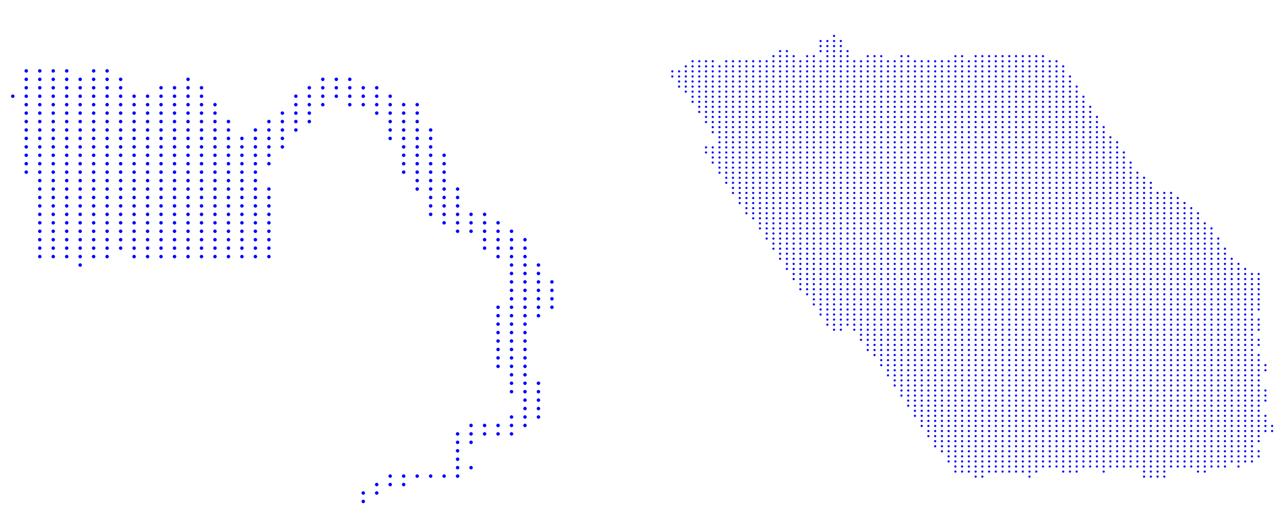
$$s(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i \phi(|\mathbf{x} - \mathbf{x}_i|) + \alpha$$

where the basis function, $\phi(r) = -\frac{1}{2\tau^3}(e^{-\tau|r|} + \tau|r|)$, is the fundamental solution to the tension differential operator, $D(\cdot) = \frac{d^4(\cdot)}{dt^4} - \tau^2\frac{d^2(\cdot)}{dt^2}$.

The coefficients are found by solving the linear system formed by the interpolation conditions, $s(\mathbf{x}_i) = f_i$, and the constraint, $\sum_{i=1}^{N} \lambda_i p(\mathbf{x}_i) = 0$, for all $p \in \Pi_{m-1}(\mathbb{R}^d)$.

Data

Bathymetric data at 100m resolution from Morro Bay and Point Buchon were used (see Figure 1). These regions each had the desired resolution and the data domain complexity to test the model.



(a) (b)

Figure: (x, y) coordinates of the data points for (a): Morro Bay and (b): Pt. Buchon Marine Protected Area (MPA).

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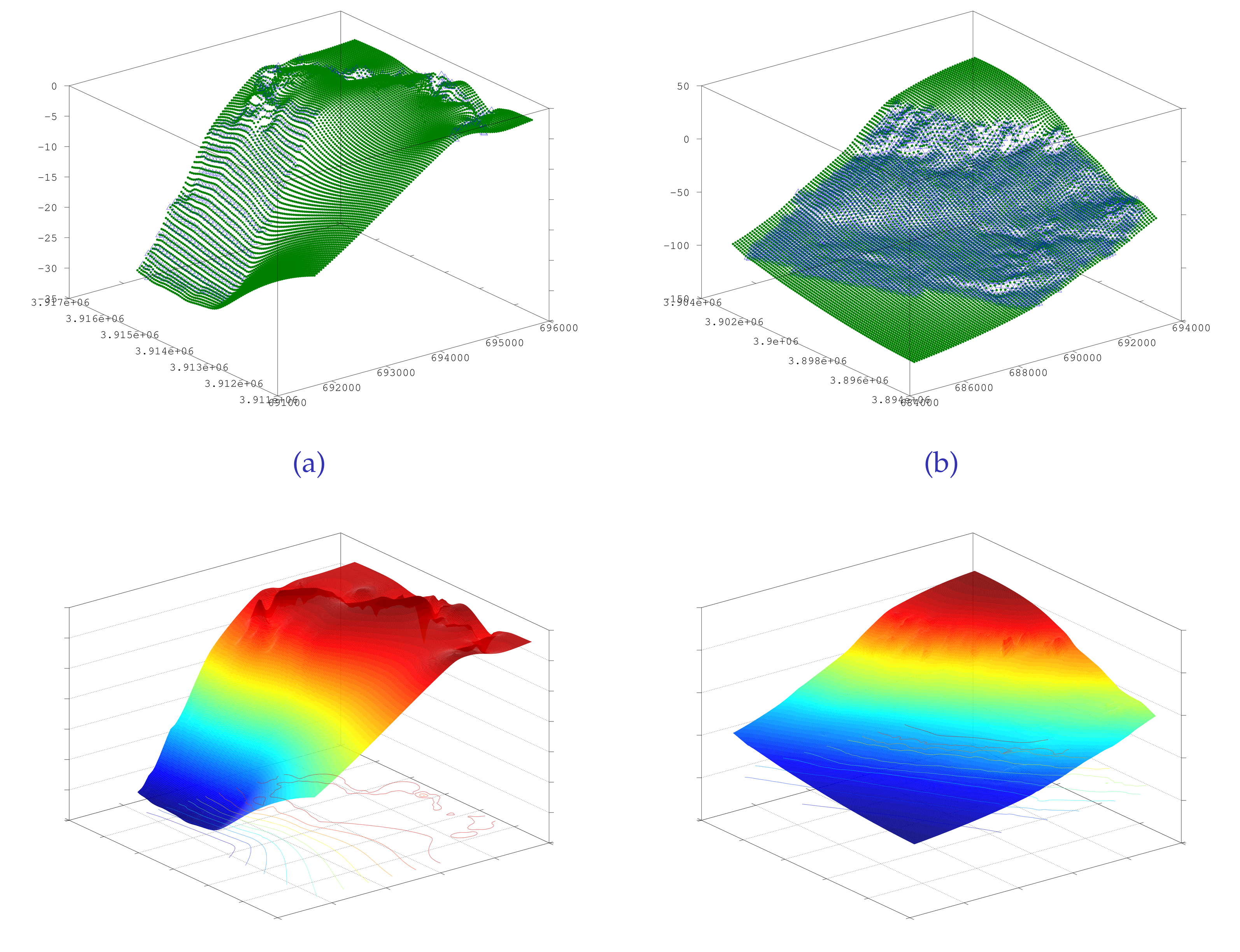
Results

Surface generated by RBFTs using 10^4 points uniformly distributed over the domain $[x_{min}, x_{max}] \times [y_{min}, y_{max}]$ and tension $\tau = 867.5309$. If the surface elevation is f, then the interpolation error is $\mathcal{O}(\tau^{-3/2}|f|)$ ([1]).



(a) (b) (c) (d)

Figure: Interpolation data points (blue) and calculated elevations (green) for (a) Morro Bay and (b) Pt. Buchon MPA. Rendered sea floor terrain for (c) Morro Bay and (d) Pt. Buchon.

Each region contained approximately 5,000 data points. The modest number of data points required allowed this simulation to be completed on a desktop computer within a few minutes.

Conclusion

RBFTs proved to be an effective basis function to interpolate terrain data measured at scattered data points. Because this method is meshfree, a pre-calculation domain tessellation stage was unnecessary resulting in decreased total processing time. This technique is easily extensible - additional data points are easily integrated into the system and do not require re-meshing the domain or calculating a new linear system.

Future Work

- Can sufficient accuracy be achieved with compactly supported RBFTs?
- How to model around an obstruction? In particular, how to image the sea floor surrounding an island?

Bibliography

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