Multiresolution Edge Detection of the Sea Floor Along the California Coast

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Abstract

A discrete wavelet transform is applied to bathymetric data taken off the California coast. Multiscale terrain features are highlighted using an edge detection algorithm. The wavelet-based method improves upon classical methods that require high-resolution data.

Motivation

The main objective of the edge detecting process is to extract accurate edge curves without changing the properties of the image. In digital terrain modeling, important features such as ridges, hills or valleys, can be extracted from the edges of an image. Wavelets are equivalent to the Canny edge detection algorithm that uses sharp changes in the modulus of the gradient vector. The multiresolution properties of wavelets, however, extracts more information. For example, it is possible to see whether it is a gradual change or a leap, a giant cliff or a momentary spike.

Characterization of Edges

Edges are characterized by their Lipschitz regularity. A function, f, is uniformly Lipschitz α over the interval (a, b) if, for every $x, y \in (a, b)$, there is a constant K such that

$$|f(x) - f(y)| \leqslant K|x - y|^{\alpha}$$

Mallat [1] showed that the if the wavelet transform is Lipschitz α, the function is also Lipschitz α

$$|W_{2^j}(f(x))| \leqslant K(2^j)^{\alpha}$$

This result can be extended to two-dimensions. A $2-D$ image is Lipschitz α in (x, y) if there is a constant K such that

$$|f(x,y) - f(x_0, y_0)| \leqslant K|(x - x_0)^2 - (y - y_0)^2|^{\alpha/2}$$

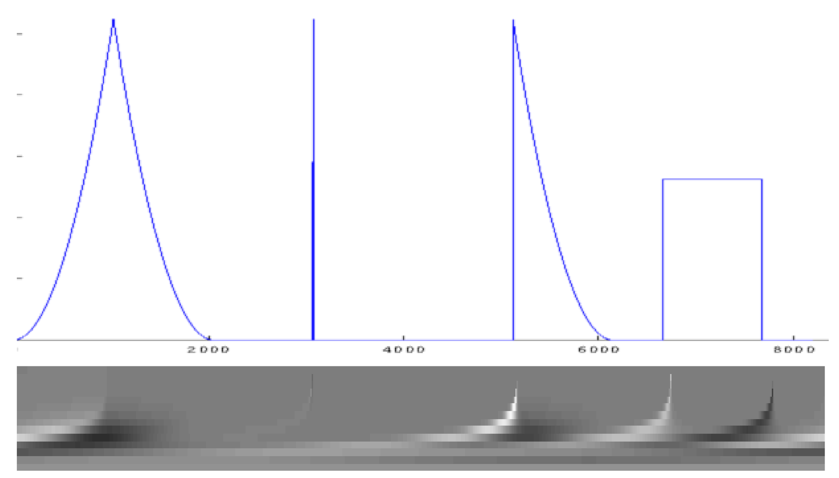

Figure: An artifical $1D$ signal to illustrate the effects of a variety of edge types on the dyadic wavelet transform. Pixel position increases from left to right and frequency increases from bottom to top. The edges in the signal result in funnel shaped patterns in the wavelet transform.

Original Data Image

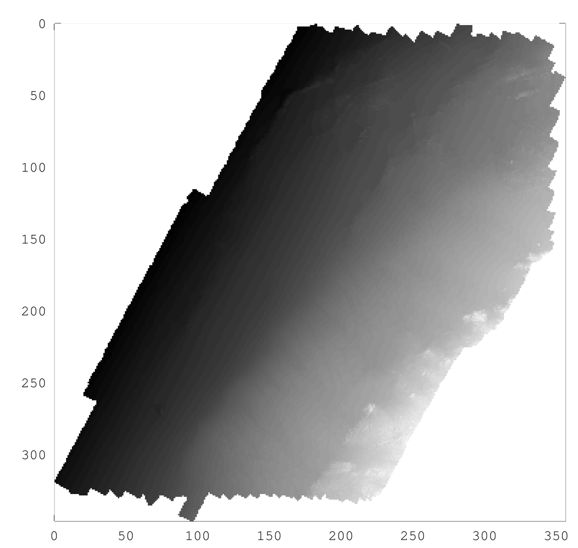

Figure: The original image of feature data at Pt. Buchon.

Modulus Maxima & Angular Image

The modulus maxima image is constructed from the horizontal & vertical components of the wavelet transform

$$M_s f(x,y) = \sqrt{|W_s^1 f(x,y)|^2 + |W_s^2 f(x,y)|^2}$$

The angular image is calculated using

$$\arctan\left(\frac{W_s^2 f(x,y)}{W_s^1 f(x,y)}\right)$$

The horizontal and vertical images form a gradient image. The modulus maxima image is a scalar value of the vector at each point and the angular velocity is the angle.

Multiresolution Imaging

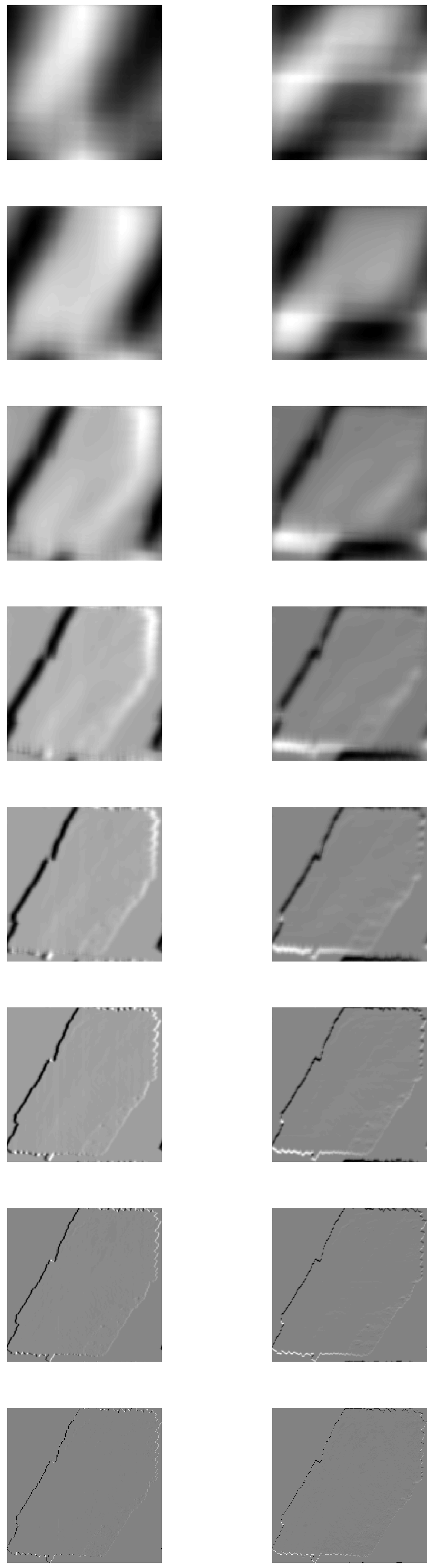

Figure: The $2^8 \times 2^8$ mesh yields 8 resolution levels. The left column corresponds to horizontal filtering and the right column corresponds to vertical filtering. The lowest resolution is along the top row and the highest is along the bottom row.

Edge Detection Algorithm

1. Apply the wavelet transform of the image. The result are two stacks of images. Since each image is $2^p \times 2^p$ pixels, p scaling levels are available for each stack. One image stack contains the horizontal filtering and the other contains the vertical filtering.
2. At each step, the image is convolved with a wavelet to obtain the coefficients at that level. It is then smoothed with a Gaussian of increasing scale.
3. Lines of maxima are found using the modulus and angular image. A pixel is a modulus maximum if it is larger than it's neighbors along the angle of the gradient vector.

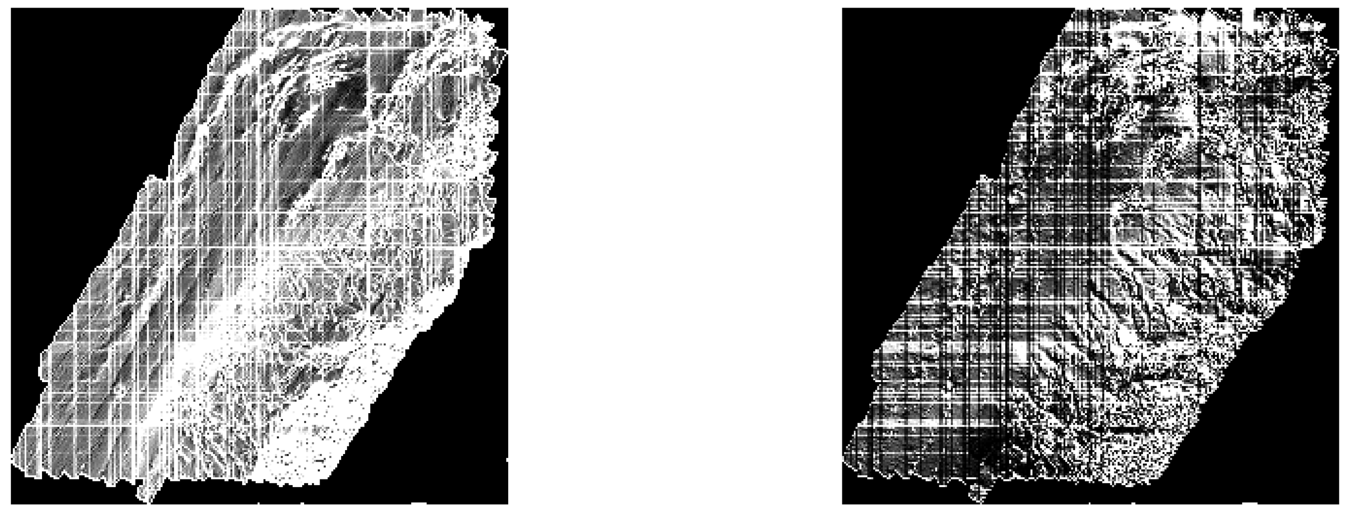

Figure: The modulus Maxima image (left) and angular image (right) corresponding to the highest resolution level. The lines on the images indicate data points that were removed to fit the required 2^p data points for the wavelet transform.

Conclusion

Edges can be classified into different types and they are characterized by their Lipschitz continuity. This continuity can be derived by observing the evolution of the wavelet transform across multiple scales. When applied to bathymetric data, wavelet-based edge detection appeared to effectively enhance terrain features.

Future Work

- The wavelet transform requires 2^p data points. Often, the resolution did not yield the ideal number of data points and we were forced to toss out data to get the desired number. Is there an effective way to extract acceptable data points?
- This approach appeared to identify the edges of the data domain. How effective is this method in identifying interfacial features; i.e. coastlines around islands?
- Radial basis functions have proven effective in modeling bathymetric data while the wavelet transform can extract terrain features. How can these methods be coupled to generate feature-rich terrain models?

Bibliography

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Academic Press, 1999.

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